EX: find the z-transform for the following seq:

$$x[n] = \left(-1/3\right)^n u[n] - \left(1/2\right)^n u[n-1]$$

SOL:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(-\frac{1}{3}\right)^n z^{-n}$$

$$- \sum_{n=-\infty}^{1} \left(\frac{1}{2}\right)^n z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(-\frac{1}{3} z^{-1}\right)^n - \sum_{n=1}^{\infty} \left(\left(\frac{1}{2}\right)^{-1} z\right)^n$$

$$= \frac{1}{1 + \frac{1}{3} z^{-1}} - \left[\frac{1}{1 + 2z} - 1\right]$$

for ROC$_1$:

$$\left|\frac{1}{3} z^{-1}\right| < 1$$

$$|z| > \frac{1}{3}$$

for ROC$_2$

$$|2z| < 1$$

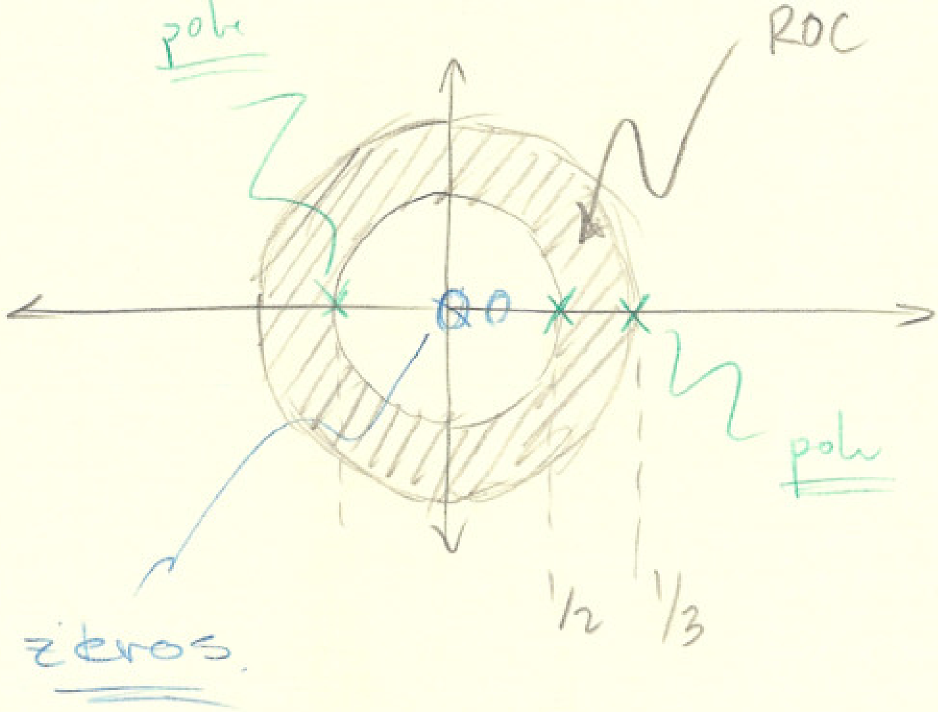
$$z < 1/2$$

$$= \frac{1}{1 + 1/3 z^{-1}} - \left[\frac{1 - 2z - 1}{1 + 2z}\right]$$

$$= \frac{1}{1 + 1/3 z^{-1}} + \frac{2z}{1 + 2z}$$

$$= \frac{1}{1 + 1/3 z^{-1}} + \frac{1}{1 + 1/2 z^{-1}}$$

$\frac{1}{3} < |z| < \frac{1}{2}$



$$= \frac{2z^2 - z/6}{(z + 1/3)(z - 1/2)}$$

poles @ $z = -1/3,\ 1/2$

zeros @ $z = 0,\ 1/12$

2.2 properties of ROC

A ROC is always bounded by a circle.

B ROC for right sided seq. is always outside a circle.

C. ROC for left sided seq. is always inside of circle.

D. ROC for both sided seq. is a ring.

E. ROC can not ~~contain~~ include any pole. There should be at least one pole at the boundry of ROC.

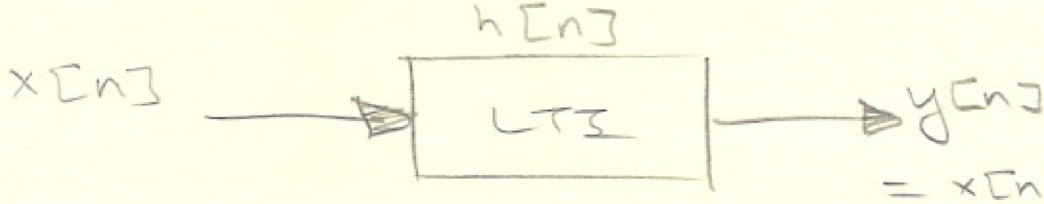
F. Must be a connected region

G. For ROC of finite length seq. is entire zplane except $z=0$ or $z=\infty$

Interpretation of z^{-1}

$X(z)$ $\qquad\qquad$ $z^{-1}X(z)$

Transfer function of an LTI system



$= x[n] * h[n]$

$X(z)$ $\qquad$ $H(z)$ $\qquad$ $Y(z)$

$= X(z) \cdot H(z)$

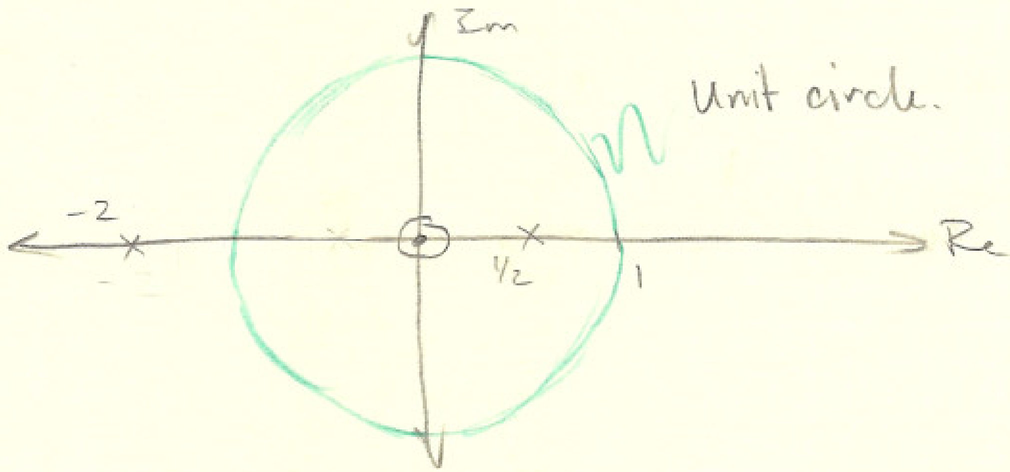
$\boxed{\text{ROC default} \quad |z| > |a|}$

by default

$h[n] = 0$ for $n < 0$

Relation b/t stability, causality & ROC

Consider the pole zero plot of the T/F, H(z) of the system is shown below.



According to the properties of ROC, there are 3 possible cases of ROC.

$\mathrm{ROC}_1 : |z| > 2$

$\mathrm{ROC}_2 : \frac{1}{2} < |z| < 2$

$\mathrm{ROC}_3 : |z| < 1/2$

If the system is stable, that means h[n] is absolutely summable. Therefor ROC must include unit circle

(ie $\mathrm{ROC} = \mathrm{ROC}_2$)

If the system is causal, h[n] should be right sided seq. Therefore ROC = ROC$_1$. Hense the system is unstable.

If ROC = ROC$_3$ the system is neither causal nor stable.

Ex $h[n] = a^n u[n]$ $|a| < 1$ for stable & causal

$$H(z) = \frac{1}{1 - az^{-1}} \quad \text{ROC} \quad |z| > |a|$$